Problem 20.48

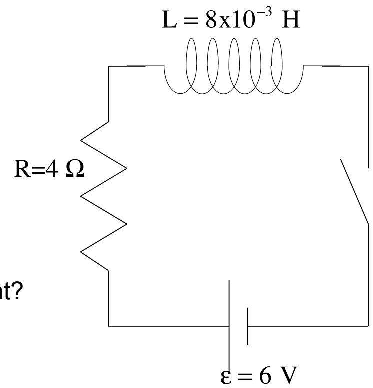
For the circuit shown, determine:

a.) what is the time constant?

b.) what is the current in the circuit after two-hundred fifty microseconds (00025 seconds)?

c.) what is the final steady-state current?

d.) how long does the current take to reach 80% of its maximum value?



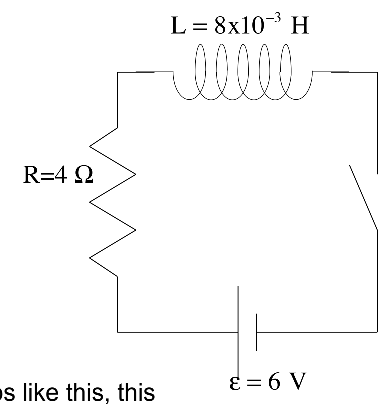
a.) what is the time constant?

$$\tau_L = L/R = \frac{(8x10^{-3}\ H)}{(4\ \Omega)} = 2x10^{-3}\ \text{seconds}$$

b.) what is the current in the circuit after $2.5x10^{-4}$ seconds?

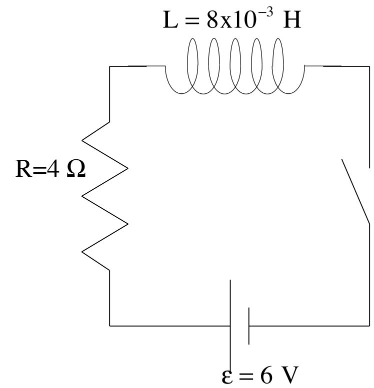
Though I don't usually use relationships like this, this is the easiest way to calculate this:

$$I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) = \frac{\varepsilon}{R}\left(1 - e^{-t/(L/R)}\right)$$



Putting in the numbers:

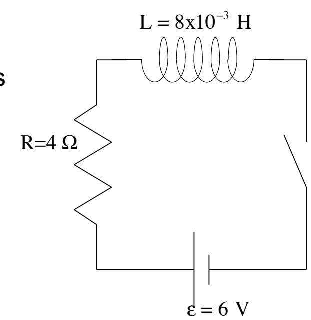
$$I = \frac{\varepsilon}{R}\left(1 - e^{-t/(L/R)}\right) = \frac{(6\ V)}{(4\ \Omega)}\left(1 - e^{-(2.5x10^{-4})/\left((8x10^{-3}\ H)/(4\ \Omega)\right)}\right) = .176\ A$$



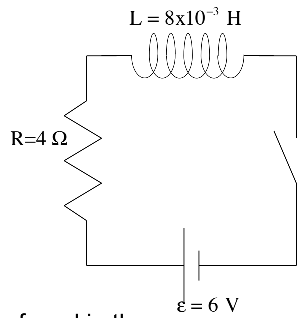
c.) what is the final steady-state current?

Once the current hits steady-state, there is no longer a change in current, which means there is no longer a changing magnetic flux through the coil (or it's associated "back EMF"). In other words, the current is what you would expect for a circuit in which there are only resistors.

$$V_R = iR \Rightarrow i = \frac{V}{R} = \frac{(6\ V)}{(4\ \Omega)} = 1.5\ A$$



d.) how long does the current take to reach 80% of its maximum value?



The general expression for the time dependent current in an RL circuit is shown below.

$$I = \frac{\varepsilon}{R}\left(1 - e^{-t/(L/R)}\right)$$

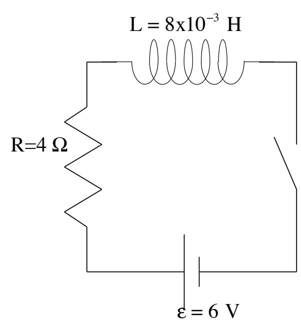
When the current is at .8i, the fractional part is found in the

$$\left(1 - e^{-t(R/L)}\right)$$

term. In other words, the time we are looking for is such that:

$$.8 = \left(1 - e^{-t(R/L)}\right)$$

Solving this, we get:



$$.8 = 1 - e^{-t(R/L)}$$

$$\Rightarrow \quad .2 = e^{-t(R/L)}$$

$$\Rightarrow \quad \ln(.2) = \ln\left(e^{-t(R/L)}\right)$$

$$\Rightarrow \quad \ln(.2) = -t\left(R/L\right)$$

$$\Rightarrow \quad t = -\left(L/R\right)\ln(.2)$$

$$\Rightarrow \quad t = -\left(\frac{(8\text{x}10^{-3}\,\text{H})}{(4\Omega)}\right)\ln(.2)$$

$$\Rightarrow \quad t = 3.3\text{x}10^{-3} \text{ seconds}$$